

The principles for appropriate pedagogy in early mathematics: Exploration, apprenticeship and sense-making: Part 2

Catherine Gripton and Helen J Williams continue their discussion of what is special about early years mathematics practice.

In this second part of our principles for appropriate pedagogy in early mathematics, we continue our reflective account of what makes early mathematics practice unique. These two articles expand upon ideas presented in issue 16 of *Impact* journal from the Chartered College of Teaching. In Part 1, we presented the first four principles that underpin mathematics teaching with birth to eight-year-olds and provided an example of these in practice. In Part 2, we share principles five to eight from the list.

As stated in Part one, early mathematical development is about exploration, apprenticeship and sense-making (ECMG, 2022). Principles 5 to 8 embrace this, considering the practitioner knowledge that informs what they do, the range of approaches they take, the opportunities they provide and the communication they encourage.

Principle 5: Early mathematics practice is best shaped by practitioner knowledge of typical developmental trajectories in mathematics

These are “descriptions of the typical path that children tend to follow in developing an understanding of a mathematical topic” (EEF, 2020: 9). They support decisions about what to offer, suggest or provide for children, now and next. They are not a curriculum map, tick-list assessment or set of sequenced objectives. Learning trajectories are knowledge of key developmental milestones about which the practitioner thinks when making decisions about what and how to teach next. They guide practitioners to notice and scaffold the mathematics concepts that will provide the firmest foundation for future mathematical learning. Clements and Sarama’s comprehensive learning trajectories offer practitioners developmental progressions for the birth to seven years age range for many areas of mathematical learning (2022).

Principle 6: Young children learn mathematics through play and need time to: (i) follow and develop their own choices and ideas, (ii) play and interact with adults, and (iii) participate in adult-led episodes

Early mathematical understanding is achieved during

both child-initiated and adult-initiated play and other meaningful contexts such as routines. All children should enjoy daily moments where they explicitly engage with mathematical concepts and language. Even play that is adult-initiated can be child-directed, allowing children to have control over what takes place following a provocation introduced by an adult. Guided play (Skene et al., 2022) can be particularly powerful, with its careful balance between adult intentions (clear focus and pedagogic flexibility) while retaining the space and time for child agency (play and choice).

Principle 7: Practitioners provide opportunities for mathematical learning through the environment and continuous provision

Adults also ensure that mathematical learning occurs in these contexts, through observation followed by sensitive interactions. Mathematics planning for children between the ages of birth and seven years begins with the deliberate choices practitioners make about the learning environment, both how this is set up and in what is provided. This is planning for the longer term. For all ages, there are also some immediate teaching decisions that are made to draw out, emphasise or extend the mathematics in-the-moment. These mathematical pedagogies can be quite subtle, such as a quizzical look or a movement of an object. They can also be more obvious, such as asking ‘I wonder if/why...?’. In between the long term and the immediate there is planning for routines such as snack-time, tidying and registration, which are authentic opportunities for mathematical learning, as are games, songs and stories. The mathematics in these contexts is meaningful to the child and provides them with a reason to problem-solve (Gripton and Williams, 2021).

Principle 8: Children have a repertoire of ways in which to communicate their mathematical thinking.

All learners, but particularly young children, have a diverse range of ways in which to communicate their mathematical thinking with others. Children represent their ideas by placing and moving objects, by creating mathematical graphics (marks, drawings and symbols)

and by using words and gestures. Practitioners use mathematical vocabulary in context, modelling gestures where appropriate, to support children's concept formation. They sensitively support children to include standard symbols in their representations when appropriate and in meaningful contexts, such as keeping score in a game. Children should have agency and ownership over their mathematical communication.

Principles 5 to 8 in practice

After enjoying the story "*Little Croc's Purse*" by Lizzie Finlay several times, the children's water play develops around 'making lemonade'. Soon there is a Lemonade Shop, with other children entering the play to buy and 'drink' the lemonade, asking for their bottle to be re-filled. Often, these over-flow and the children seem not very interested in questions about how full the bottles are when asked by an adult. The practitioner plays alongside the children and notices that children seem to be paying attention to the container that is being emptied, rather than the one being filled. She decides to make some changes to the provision. She sorts out a range of transparent

containers that a child can lift in one hand and some jugs for easy pouring and places a table nearby on which the filled containers can be placed. She colours the water in order that the contents can be easily seen and, importantly, sticks a band of masking tape around each container, which is to be the 'fill up to here' line.

After demonstrating carefully pouring from the jug to the 'fill up to here' lines and asking children to call out when the 'up to' line is reached, the practitioner leaves the children to play with this idea. Children independently practise careful lemonade pouring, taking it in turns to hold their chosen container whilst the other fills it with a jug of 'lemonade'. The child holding the container watches it fill in order to call out "stop!" when the tape line is reached. The bottles of lemonade are lined up on the table. Now more effective discussion takes place about whether the containers are filled 'fairly', not full enough, fuller than others, 'nearly full', or too full to carry (or 'drink'). Later, children decide which prices are charged for different amounts of lemonade.

Principles	Practice
5. Early mathematics practice should be shaped by practitioner knowledge of the typical developmental trajectories in mathematics	The practitioner knows that comparison is an important element of measurement and that the children's emptying and filling of containers is not currently furthering their understanding of capacities. The practitioner has adapted provision to encourage children's precision in comparing different amounts of water. The practitioner is aware of the need to extend the children's thinking from 'full' and 'empty' to comparative 'more/less full than' (using their knowledge of developmental trajectories).
6. Young children learn mathematics through play and need time to: (i) follow and develop their own choices and ideas, (ii) play and interact with adults, and (iii) participate in adult-led episodes	The practitioner read a story (adult input) which the children chose to take into their water play (child-led), The practitioner observed this and adapted the activity and interacted with the water play to teach children how to measure with increased precision (guided play where adults interact and participate in children's play to develop mathematical ideas further).
7. Practitioners provide opportunities for mathematical learning through the environment and continuous provision	The water tray is always available to the children as part of the outdoor continuous provision and on this occasion is being used by the children (and by the practitioner) for a specific mathematical purpose.
8. Children need to have a repertoire of ways in which to communicate their mathematical thinking.	The practitioner models language and gestures by asking children to check it is at the 'up to line' (hand up) and models the language 'nearly as full as' and 'fuller than'. The children adopt some of these to share their thinking through their play. Children are encouraged to reflect on and share this in class discussions.

Table 1



This example is a mathematical moment from the Early Childhood Mathematics Group (ECMG). It describes a practitioner's approach to following children's interests whilst developing children's mathematical learning using their practitioner knowledge. Table 1

explains how we see principles 5 to 8 in this practice example.

We suggest that the eight research-informed principles underpin appropriate early mathematics pedagogy. They are sufficiently flexible to apply across the diverse range of settings within the age range birth to seven. We feel sure that these principles are sufficiently well informed to stand the test of time. New research, guidance and frameworks help us to develop specific resources and approaches, but the principles for appropriate pedagogy can help to guide the ways in which these are implemented in practice to most effectively support young learners to be confident, determined, creative and joyful mathematicians.

(Photograph courtesy of Esther O'Connor)

Principles for appropriate pedagogy in early mathematics

1. All children are entitled to be mathematical and learn mathematics
2. Mathematical development involves learning mathematical behaviours (or ways of working mathematically) as well as mathematical concepts
3. Young children are differently experienced and not differently able at mathematics
4. Children are entitled to pedagogy that supports the full breadth of mathematical learning
5. Early mathematics practice should be shaped by practitioner knowledge of the typical developmental trajectories in mathematics
6. Young children learn mathematics through play and need time to: (i) follow and develop their own choices and ideas, (ii) play and interact with adults, and (iii) participate in adult-led episodes
7. Practitioners provide opportunities for mathematical learning through the environment and continuous provision
8. Children need to have a repertoire of ways in which to communicate their mathematical thinking.

The Mathematical Moment is adapted with permission from Helen J Williams' book: 'Playful Mathematics with 3 to 7-year-old children' (Sage, 2022) and available here: <https://earlymaths.org/wp-content/uploads/2022/06/Mathematical-moments-34s-measures.pdf>

References

Clements D.H. and Sarama, J. (2022) *Learning and teaching with learning trajectories*. <https://www.learningtrajectories.org/>

Early Childhood Mathematics Group (ECMG) (2022) *Key principles: Early childhood mathematics pedagogy: Exploration, apprenticeship and making sense*. Available at: <https://earlymaths.org/early-years-mathematics-pedagogy-exploration-apprenticeship-making-sense>

Education Endowment Foundation (EEF) (2020) *Improving mathematics in the Early Years and Key Stage 1: Guidance report*. Available at: https://d2tic4wvo1iusb.cloudfront.net/eef-guidance-reports/early-maths/EEF_Maths_EY_KS1_Guidance_Report.pdf

Gripton C. and Williams H.J. (2021) Harnessing the power of story to promote mathematical learning. In: *Primary Education Network*. Available at: <https://blogs.nottingham.ac.uk/primaryeducationnetwork/2021/12/16/harnessing-the-power-of-story-to-promote-mathematical-learning>

Skene K., O'Farrelly C., Byrne E., Kirby, N., Stevens, E. and Ramchandami, P. (2022) Can guidance during play enhance children's learning and development in educational contexts? A systematic review and meta-analysis. *Child Development*. <https://srcd.onlinelibrary.wiley.com/doi/10.1111/cdev.13730>

The attached document has been downloaded or otherwise acquired from the website of the Association of Teachers of Mathematics (ATM) at www.atm.org.uk

Legitimate uses of this document include printing of one copy for personal use, reasonable duplication for academic and educational purposes. It may not be used for any other purpose in any way that may be deleterious to the work, aims, principles or ends of ATM. Neither the original electronic or digital version nor this paper version, no matter by whom or in what form it is reproduced, may be re-published, transmitted electronically or digitally, projected or otherwise used outside the above standard copyright permissions. The electronic or digital version may not be uploaded to a website or other server.

Any copies of this document MUST be accompanied by a copy of this page in its entirety. If you want to reproduce this document beyond the restricted permissions here, then application must be made for express permission to copyright@atm.org.uk. The exception to the above is for the original author(s) who retain individual copyright.

ATM is a not for profit professional teaching association. The majority of funding used to produce and prepare the MT journal is procured through our membership subscriptions.



Mathematics Teaching does not seek to conform to an 'official' view on the teaching of mathematics, whatever that may be. The editorial board wishes to encourage contributors to express their personal views on the teaching and learning of mathematics.

ATM is an association of teachers in which everyone has a contribution to make, experiences and insights to share. Whether practical, political, philosophical or speculative, we are looking for articles which reflect on the practice of teaching mathematics. We aim to publish articles that will be of interest to the breadth of our membership, from the Foundation Stage to Higher and Further Education; as well as a balance between those derived from research and from practical experience. Submitted articles are accepted for publication based on their clarity, topicality, the extent to which they reflect upon knowledge and understanding of mathematics teaching and learning, and their contribution to inspiring further development and research.



Join ATM at any time and receive twelve months of membership, including instant access to member discounts and resources. Spread the cost and pay in ten monthly instalments.

Membership Includes:

- Five copies of the ATM journal Mathematics Teaching (MT)
- A 25% discount on all shop items
- Considerable discounts at the hugely popular annual ATM conference
- Electronic access to thousands of online MT journal articles
- Access to all online member-only resources
- Professional support and enrichment – being part of a community where ideas are generated and shared
- Regular ATM e-newsletters, containing current news and activities
- A network of local branches offering regular meetings
- Accreditation - ATM is proud to offer members the opportunity to apply for the CMathTeach Designation, making ATM membership the route to Chartered Mathematics Teaching status
- Influence and having a voice - eligibility to vote on resolutions that shape the direction of ATM

Join ATM Today